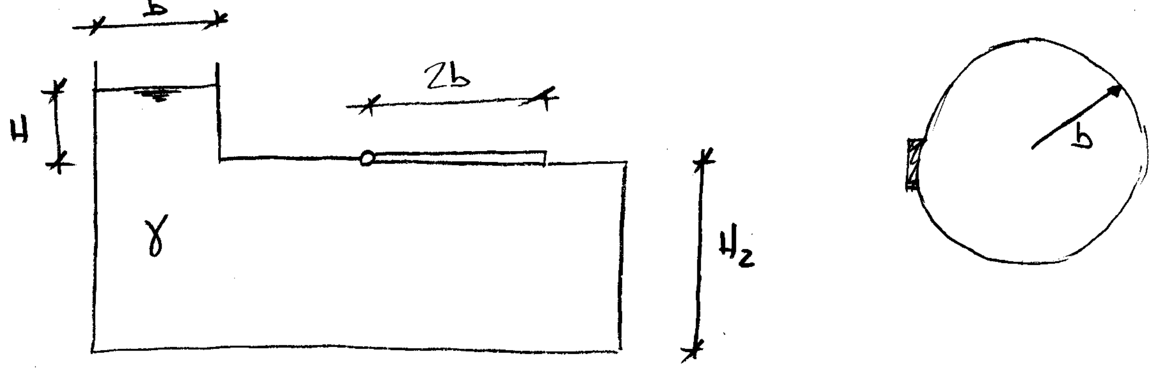
IDROSTATICA

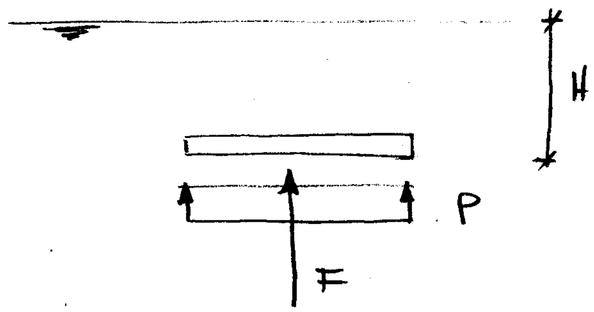
ESERCIZI SVOLTI da PEDRIZZETTI

n° 1



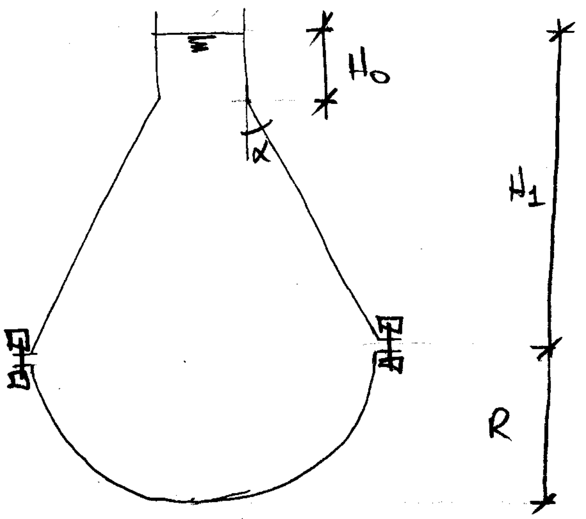
Pb: Calcolare quanto deve pesare il portello affinché l'acqua non esca.

Il problema fondamentale è quello di modellizzare:

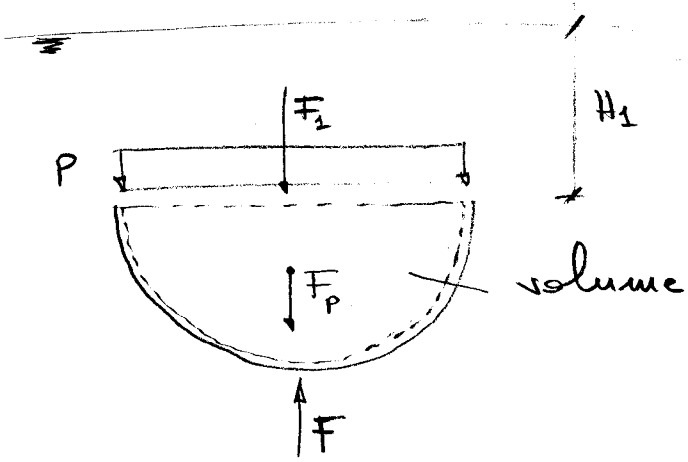


$$F = p \cdot A = \gamma H \cdot \pi b^2$$

n° 2



Pb: calcolare la forza agente su di 1 degli 8 bulloni che tengono fissata la calotta emisferica.



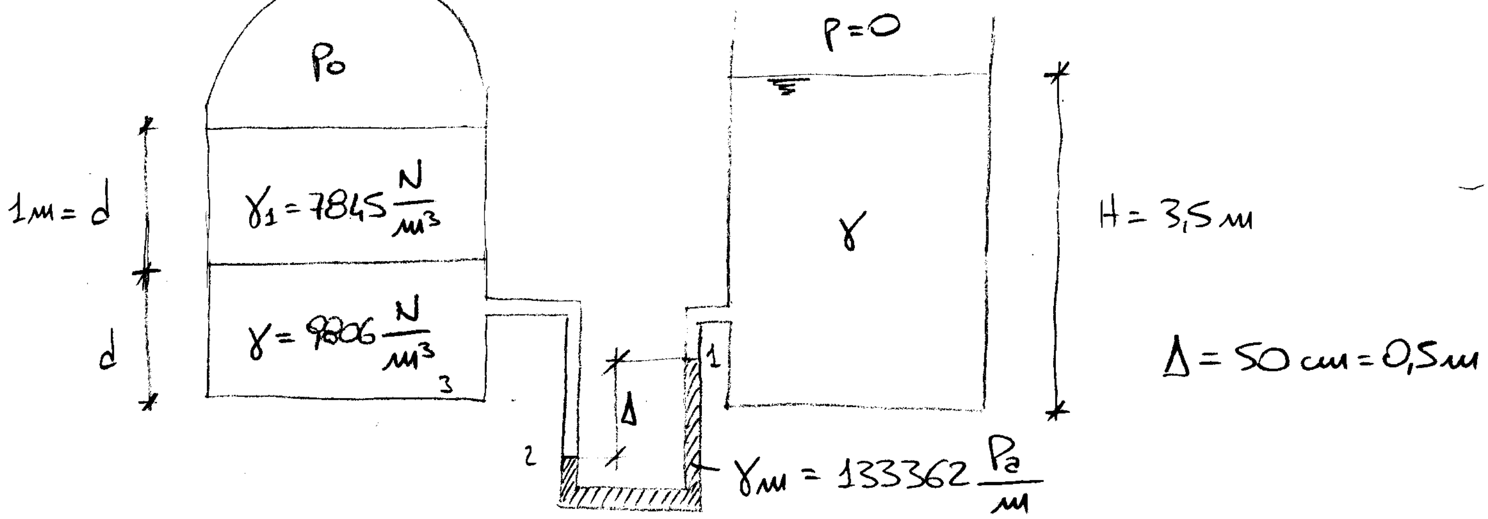
$$F_1 = p \cdot A = \gamma H_1 \cdot \pi R^2$$

$$F_p = \gamma \cdot V = \gamma \cdot \frac{1}{2}\left(\frac{4}{3}\pi R^3\right) = \gamma \frac{2}{3}\pi R^3$$

$$F = F_1 + F_p = \gamma\left(H_1 \pi R^2 + \frac{2}{3}\pi R^3\right) = \gamma \pi R^2\left(H_1 + \frac{2}{3}R\right)$$

$$F_b = \frac{1}{8}F = \frac{\pi}{8}\gamma R^2\left(H_1 + \frac{2}{3}R\right)$$

n° 3



Pb: calcolare p_0.

① calcoliamo la differenza tra le pressioni p_1 e p_2

$$h_1 = h_2 \Rightarrow z_1 + \frac{p_1}{\gamma_m} = z_2 + \frac{p_2}{\gamma_m}$$

$$\Rightarrow p_2 - p_1 = (z_1 - z_2)\gamma_m = \Delta \gamma_m = 66681\ Pa$$

② calcoliamo i carichi piezometrici dell'acqua dalle due parti:

$$h_1 = H = 3{,}5\,m = z_1 + \frac{P_1}{\gamma}$$

$$h_2 = z_2 + \frac{P_2}{\gamma}$$

$$h_1 - h_2 = (z_1 - z_2) + \frac{P_1 - P_2}{\gamma} = \Delta + \frac{-(P_2 - P_1)}{\gamma}$$

$$= 0{,}5\,m - \frac{66681\,Pa}{9806\,\frac{Pa}{m}} = 0{,}5\,m - 6{,}8\,m = -6{,}3\,m$$

$$h_2 = h_1 - (h_1 - h_2) = H - (-6{,}3\,m) = 9{,}8\,m$$

③

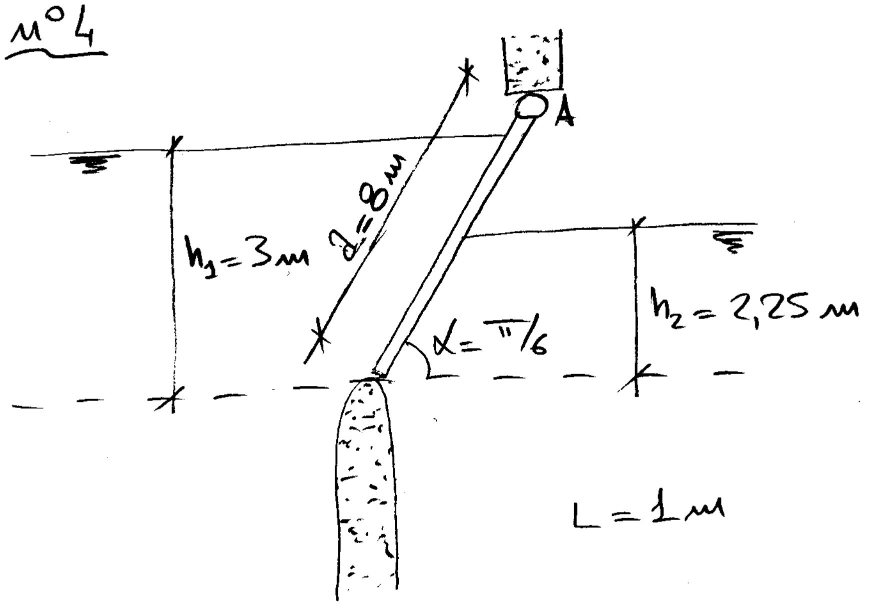
$$\begin{cases} h_2 = z_3 + \frac{P_3}{\gamma} \\ z_3 = 0 \end{cases} \Rightarrow P_3 = h_2 \cdot \gamma = 9{,}8\,m \cdot 9806\,\frac{Pa}{m} = 96098{,}8\,Pa$$

$$P_3 = P_0 + \gamma_1 d_1 + \gamma d_2$$

$$\Rightarrow P_0 = P_3 - \gamma_1 d_1 - \gamma d_2 = 96098{,}8\,Pa - 7845\,\frac{Pa}{m} \cdot 1\,m - 9806\,\frac{Pa}{m} \cdot 1\,m$$

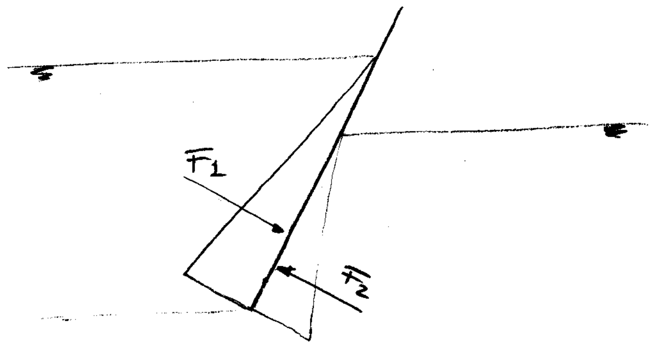
$$= 78447{,}8\,Pa$$

<u>n° 4</u>



Pb: calcolare il momento M_A da applicare alla cerniera della paratoia perché stia ferma.

E' necessario calcolare le forze ed i punti di applicazione da ambo le parti:



$$F_1 = P_{G_1} \cdot A_1 = \left(\gamma \cdot \frac{h_1}{2}\right)\left(\frac{h_1}{\operatorname{sen}\alpha} L\right) = 9806 \frac{N}{m^3} \cdot 1{,}5\,m \cdot \frac{3\,m \cdot 1\,m}{\frac{1}{2}}$$

$$\stackrel{!}{=} 88254\,N$$

$$F_2 = P_{G2} \cdot A_2 = \left(\gamma \frac{h_2}{2}\right)\left(\frac{h_2}{\operatorname{sen}\alpha} L\right) = \gamma \frac{h_2^2}{2\operatorname{sen}\alpha} L$$

$$\stackrel{!}{=} 9806 \cdot \frac{(2{,}25\,m)^2}{2 \cdot \frac{1}{2}} \cdot 1\,m = 49643\,N$$

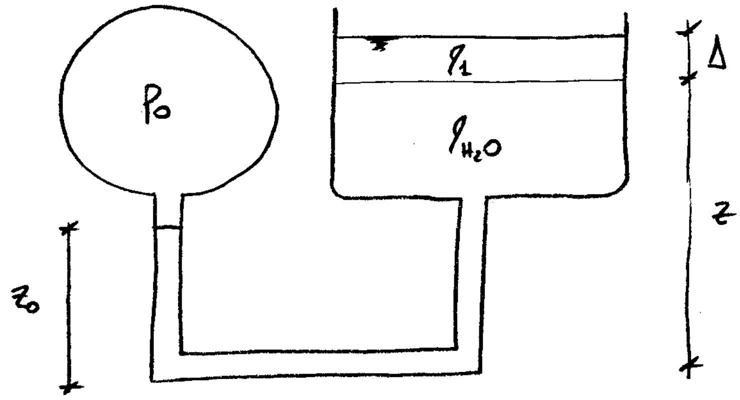
$$M_1 = F_1 \cdot \left(a - \frac{1}{3}\frac{h_1}{\operatorname{sen}\alpha}\right) \quad \text{antiorario } (+)$$

$$M_2 = F_2 \cdot \left(a - \frac{1}{3}\frac{h_2}{\operatorname{sen}\alpha}\right) \quad \text{orario } (-)$$

$$M_A = M_1 - M_2 = 88254\,N \cdot 6\,m - 49643 \cdot 6{,}5\,m$$

$$\stackrel{!}{=} 206845\,J$$

n° 5



$\rho_1 = 750 \ \frac{kgm}{m^3}$

$\Delta = 20 \text{ cm} = 0{,}2 \text{ m}$

$P_0 = 10 \text{ kPa}$

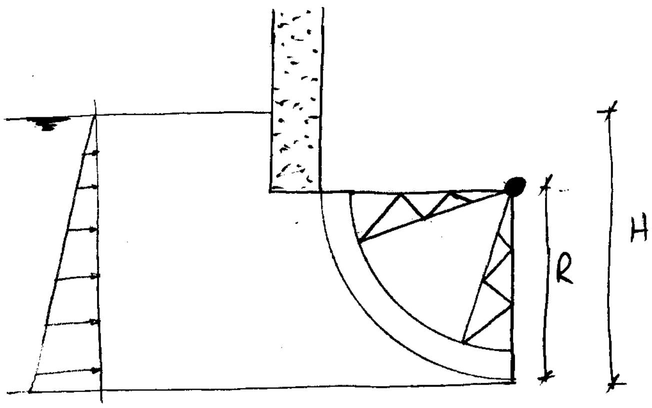
$z_0 = 70 \text{ m}$

$z = ?$

$$\gamma_1 = \rho_1 \cdot g = 750 \ \frac{kgm}{m^3} \cdot 9{,}81 \ \frac{m}{s^2} = 7358 \ \frac{N}{m^3}$$

$$\begin{cases} z_0 + \dfrac{P_0}{\gamma} = z + \dfrac{P}{\gamma} \\ P = \gamma_1 \Delta \end{cases} \Rightarrow \quad z = z_0 + \frac{P_0 - \gamma_1 \Delta}{\gamma} = 70 \text{ m} + \frac{10^4 \text{ Pa} - (7358 \cdot 0{,}2) \text{ Pa}}{9806 \ \frac{N}{m^3}} = 70{,}87 \text{ m}$$

n° 6

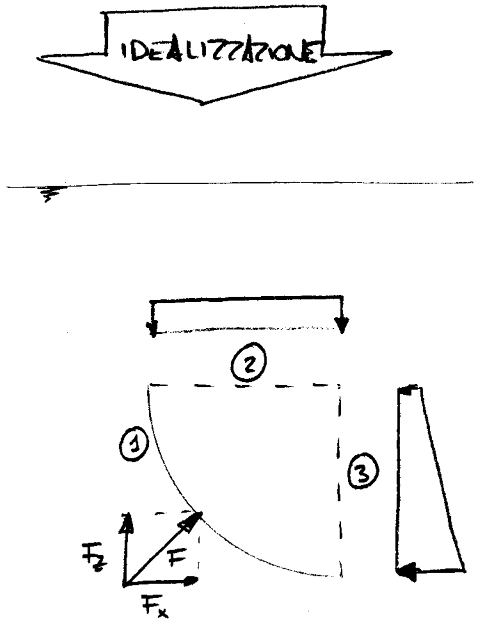


Determinare le componenti della forza agenti sulla paratia:

$H = 20 \text{ m}$

$R = 3 \text{ m}$

$(L = 1 \text{ m})$



$\underline{G} + \underline{\Pi} = 0$

$x>$ $\quad G_x = 0$

$\Pi_x = \Pi_{x_3} = p_{G3} \cdot A_3 = -\gamma \left(H - \frac{R}{2}\right) R$

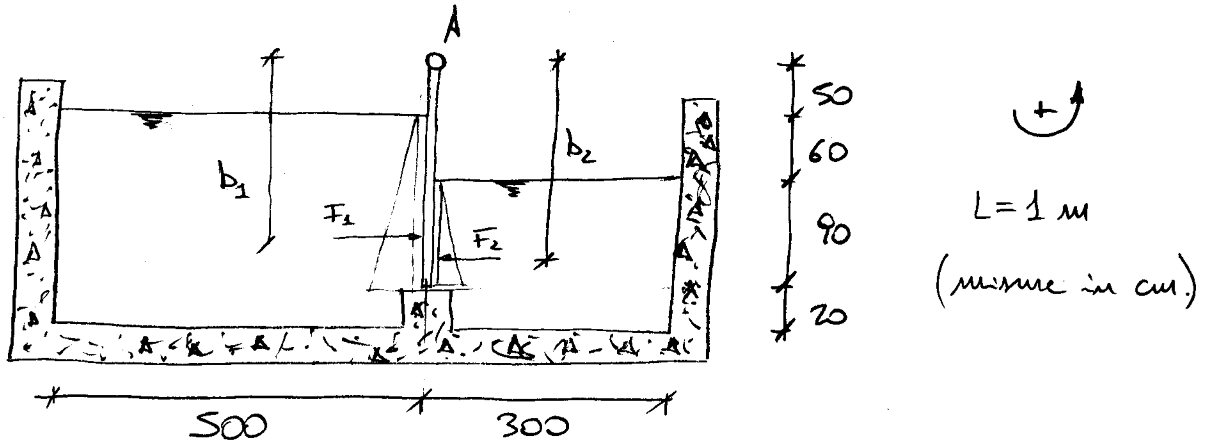
$z>$ $\quad G_z = -\gamma V = -\gamma \left(\frac{\pi}{4} R^2\right)$

$\Pi_z = \Pi_{z2} = p_{G2} \cdot A_2 = -\gamma (H - R) R$

$\Rightarrow F_x = -\Pi_{x_3} = \gamma R \left(H - \frac{R}{2}\right) \overset{!}{=} 9806 \cdot 3 \, (20 - 1{,}5) = 544{,}233 \text{ kN}$

$\Rightarrow F_z = -(G_z + \Pi_{z2}) = +\gamma \left(\frac{\pi}{4} R^2 + R(H-R)\right) \overset{!}{=} 9806 \left(3^2 \frac{\pi}{4} + 3 \cdot 17\right) = 569{,}418 \text{ kN}$

$n°7$



Calcolare il momento agente sulla cerniera A.

$$F_1 = P_{G1} \cdot A_1 = \gamma \left(\frac{0{,}6+0{,}9}{2}\right)(0{,}6 \cdot 0{,}9) = 9806 \cdot \frac{1{,}5^2}{2} = 11{,}032 \text{ kN}$$

$$b_1 = 0{,}5 + \frac{2}{3}(60+90) = 1{,}5 \text{ m}$$

$$\Rightarrow M_1 = F_1 \cdot b_1 = 16{,}548 \text{ kJ}$$

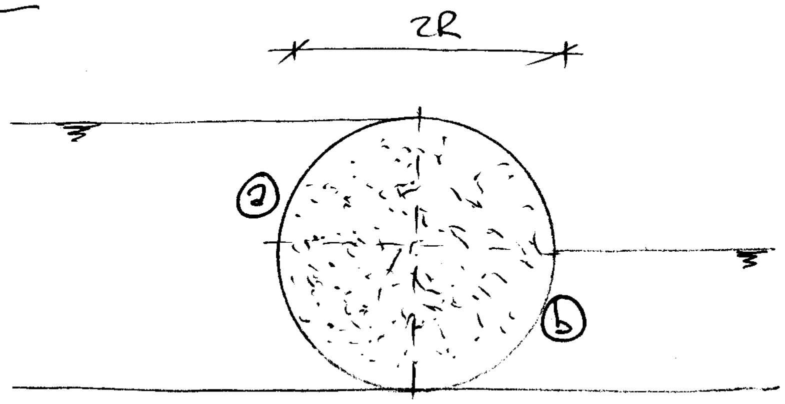
$$F_2 = P_{G2} \cdot A_2 = \gamma \frac{0{,}9}{2} 0{,}9 = 9806 \cdot \frac{0{,}9^2}{2} = 3{,}971 \text{ kN}$$

$$b_2 = 0{,}5 + 0{,}6 + \frac{2}{3} 0{,}9 = 1{,}7 \text{ m}$$

$$\Rightarrow M_2 = -F_2 \cdot b_2 = -6{,}751 \text{ kJ}$$

$$M_A = M_1 + M_2 = 11032 - 6751 = 4281 \text{ kJ}$$

$n°8$



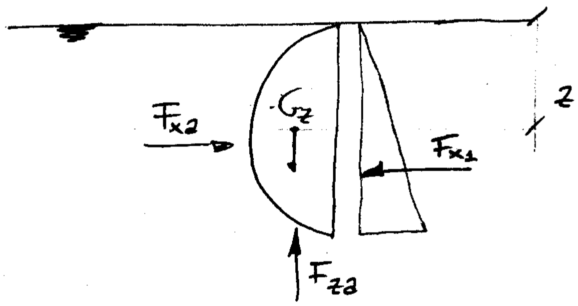
R = 18 cm = 0,18 m

L = 2 m

Trovare le componenti delle forze agenti sul cilindro.

Possiamo visualizzare il problema in due parti distinte a e b

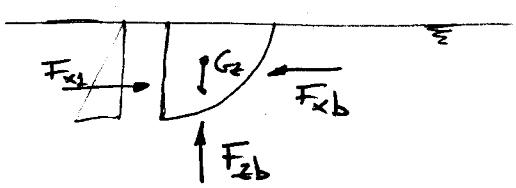
ⓐ



$$F_{xa} = F_{x1} = \gamma R \cdot 2RL = 2\gamma R^2 L = 2 \cdot 9806 \cdot 0{,}18^2 \cdot 2 = 1271\ N$$

$$F_{za} = G_z = \gamma V = \gamma \cdot \frac{\pi R^2}{2} \cdot L = 9806 \cdot \frac{\pi \cdot 0{,}18^2}{2} \cdot 2 = 998\ N$$

ⓑ



$$F_{xb} = F_{x1} = \gamma \cdot \frac{R}{2} \cdot RL = 9806 \cdot \frac{0{,}18^2}{2} \cdot 2 = 318\ N$$

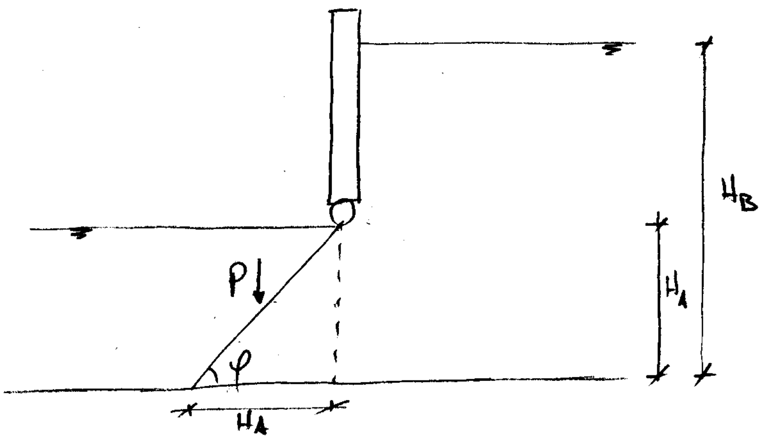
$$F_{zb} = G_z = \gamma V = \gamma \cdot \frac{\pi R^2}{4} \cdot L = 9806 \cdot \frac{\pi \cdot 0{,}18^2}{4} \cdot 2 = 499\ N$$

Avremo così sull'intero cilindro:

$$F_x = F_{xa} - F_{xb} = 1271 - 318 = 953\ N$$

$$F_z = F_{za} + F_{zb} = 998 + 499 = 1497\ N$$

n° 9



$H_A = 2m$

$H_B = 4m$

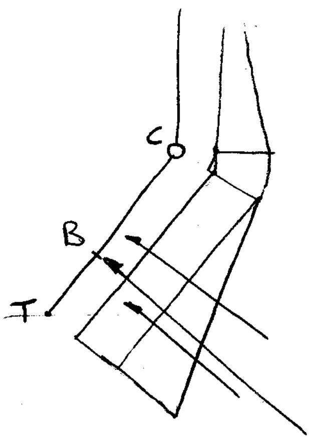
$\varphi = \frac{\pi}{2}$

Calcolare il peso P della paratoia perché essa stia ferma.

$$F_A = \gamma \cdot \frac{H_A}{2} \cdot \frac{H_A}{\cos\varphi} = 9806 \cdot \frac{2}{2} \cdot \frac{2}{\frac{\sqrt{2}}{2}} = 9806 \cdot \frac{4}{\sqrt{2}} = 27736\ N$$

$$b_a = \frac{2}{3} \frac{H_A}{\cos\varphi} = \frac{2}{3} \cdot \frac{2}{\frac{\sqrt{2}}{2}} = \frac{4}{3}\sqrt{2} = 1{,}886\ m$$

$$F_B = \gamma \left(H_B - \frac{H_A}{2}\right) \frac{H_A}{\cos\varphi} = 9806 \cdot \left(4 - \frac{2}{2}\right) 2\sqrt{2} = 9806 \cdot 3 \cdot 2\sqrt{2} = 83207\ N$$



$$\xi_B = \frac{2}{3} \frac{\xi_T^3 - \xi_C^3}{\xi_T^2 - \xi_C^2} = \frac{2}{3} \frac{4^3 - 2^3}{4^2 - 2^2}$$

$$= \frac{2}{3} \frac{64 - 8}{16 - 4} = \frac{\cancel{2}}{3} \cdot \frac{56}{\cancel{12}_6} = \frac{28}{9} = 3{,}111\ м$$

$$b_B = \left[\xi_C - (H_B - H_A)\right] \cdot \frac{1}{\cos\alpha} = (3{,}111 - 2)\sqrt{2}$$

$$= 1{,}571\ м$$

$$P \cdot \frac{H_A}{2} + F_A b_A - F_B b_B = 0$$

$$P = \frac{-27736 \cdot 1{,}886 + 83207 \cdot 1{,}571}{1} = 78408\ N$$